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J80-134

Radiative Transfer of Energy in a Cylindrical Enclosure with Heat Generation

60002
90005

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Nomenclature

B = emissive power, $\sigma_s T^4$
Q = radiative heat flux
ℓ = direction cosine
S = heat source per unit volume
T = temperature
U = dimensionless heat generation rate per unit volume, nondimensionalized by $\alpha\psi^*$
α = volumetric absorption coefficient
ϵ = emissivity of the medium
ϵ_w = emissivity of the boundaries
λ = $\sigma/(\alpha+\sigma)$
σ = volumetric scattering coefficient
τ = $(\alpha+\sigma)r$
σ_s = Stefan-Boltzmann constant
Φ = dimensionless emissive power, nondimensionalized by $\pi\psi^*$
ψ = dimensionless intensity of radiation, nondimensionalized by ψ
ψ^* = reference intensity of radiation
Ω = solid angle, 4π

Subscripts

w = wall
1 = boundary
2 = outer boundary
r,θ,z = directions in the cylindrical coordinate system

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Introduction

TO overcome the inaccuracy of the P_1 approximation of the spherical harmonics method in optically thin regions, various modifications and improvements have been introduced. Olfe[1] obtained good results by analyzing the surface emissive power contribution exactly while utilizing P_1 approximation for the gas emission contribution. Modest and Stevens[2] used P_1 approximation and exact treatment to obtain geometric factors which improve the results. Chou and Tien[3] approached the problem by approximating the moments of intensity by their mean values and dividing the radiation field into solid angle subregions which accounts for the curvature effects and give better results. Yuen and Tien[4] obtained quite accurate results through their analysis by subdividing the concentric cylindrical enclosure into many cylindrical subsections. However, the analysis may become complex if the modifications are extended to multidimensional problems. The higher order approximations of the spherical harmonics method would retain the differential nature for multidimensional problems and may result in more accurate solutions than P_1 approximations. In plane and spherical symmetric geometries, the differential approximations as high as P_{20} have been studied by Schmidt and Gelbard[5] and Federighi.[6] In cylindrical geometry P_3 is the highest order approximation that has been recently analyzed by Bayazitoglu and Higenyi[7] for the radiative equilibrium problems of a nonscattering medium. In the present work, the higher order differential approximations, the P_1, P_3, and P_5 approximations of the spherical harmonics method, are used to study the radiative transfer in cylindrical symmetry systems.

Analysis

The intensity of radiation ψ is expanded in a series of normalized spherical harmonics. This expansion is expressed in terms of the moments of intensity. The P_n approximation follows when one terminates the series after certain number of terms. For the cylindrical symmetry problems (one-dimensional), the P_5 approximation will result in the following intensity of radiation profile

$$\pi\psi(r,\beta,\phi) = \tfrac{1}{4}\psi_0 + \tfrac{3}{4}\psi_r \sin\beta\cos\phi + \tfrac{5}{8}(3\psi_{rr} - \psi_0) \times \sin^2\beta\cos 2\phi + \tfrac{7}{8}(5\psi_{rrr} - 3\psi_r)\sin^3\beta\cos 3\phi + 9/32(35\psi_{rrrr} - 30\psi_{rr} + 3\psi_0)\sin^4\beta\cos 4\phi + 11/32(63\psi_{rrrrr} - 70\psi_{rrr} + 15\psi_r)\sin^5\beta\cos 5\phi \tag{1}$$

and the coordinate system is shown in Fig. 1. The P_1 and P_3 approximations correspond to the first two terms and the first seven terms of Eq. (1), respectively. The moments used in Eq. (1) are called the zeroth, first, second, and so on.

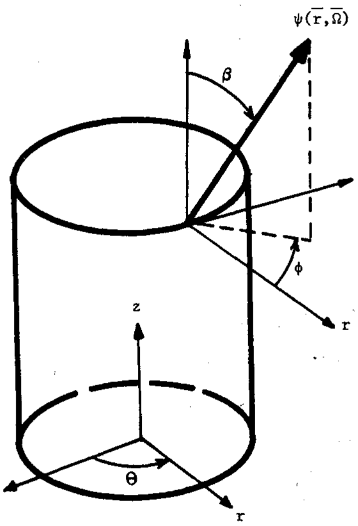

Fig. 1 Coordinate system for cylindrical geometry.

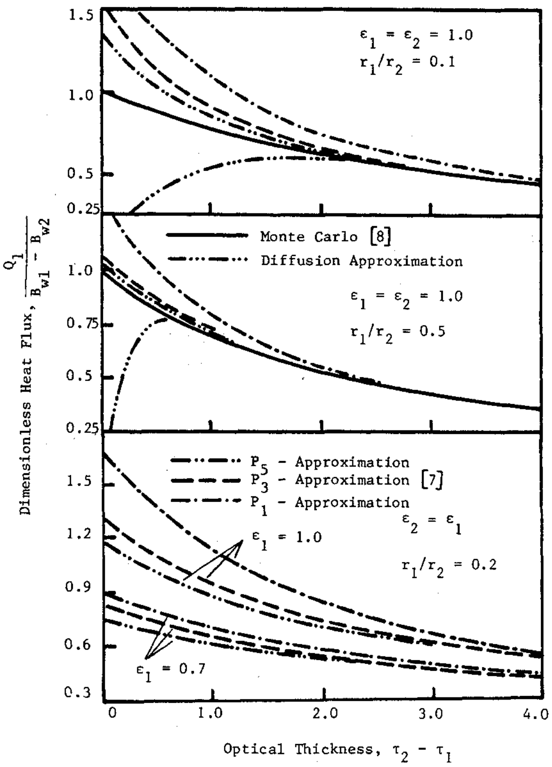

Fig. 2 Heat flux between infinitely long concentric cylinders.

Using the definition of zeroth order moment of intensity, ψ_0 the radiative transport equation for an isotropically scattering gray medium at steady state and local thermodynamic equilibrium reduces to

$$\ell_r \frac{\partial \psi}{\partial \tau} - \frac{\ell_\theta}{\tau}\frac{\partial \psi}{\partial \phi} + \psi = (1-\lambda)\phi + \frac{\lambda}{4\pi}\psi_0 \tag{2}$$

The moment differential equations are developed through multiplying Eq. (2) by the powers of ℓ_r and integrating over solid angle 4π. For the P_5 approximation, there will be six moment differential equations. The closure condition is obtained by substituting the approximate radiation intensity profile given by Eq. (1) into the definition of sixth moment of intensity, which is

$$\psi_{rrrrrr} = \frac{15}{11}\psi_{rrrr} - \frac{5}{11}\psi_{rr} + \frac{5}{231}\psi_0 \tag{3}$$

In the present work, the medium will exchange heat by radiation and it is subjected to internal heat generation. Therefore, the conservation of thermal energy reduces to

$$U = 4\pi\Phi - \psi_0 \tag{4}$$

Having the moment differential equations and introducing the proper closure condition for the approximation to be used, and stating the conservation of thermal energy of the problem, this system of governing equations can be solved if the heat generation rate and the appropriate boundary conditions are specified. However, it is usually more convenient to reduce this set of first order differential equations to one differential equation of higher order. This is achieved by differentiation and backward substitution. The differential equation is obtained for P_5 approximation is

$$\begin{aligned}
&\frac{d^6\psi_0}{d\tau^6} + \frac{67}{15\tau}\frac{d^5\psi_0}{d\tau^5} - \left(\frac{294}{25} + \frac{13}{15\tau^2}\right)\frac{d^4\psi_0}{d\tau^4} - \left(\frac{7246}{225\tau} - \frac{8}{5\tau^3}\right) \\
&\times \frac{d^3\psi_0}{d\tau^3} + \left(\frac{77}{5} + \frac{692}{225\tau^2} - \frac{14}{5\tau^4}\right)\frac{d^2\psi_0}{d\tau^2} + \left(\frac{77}{5\tau} - \frac{692}{225\tau^3} + \frac{14}{5\tau^5}\right) \\
&\times \frac{d\psi_0}{d\tau} = \frac{231}{5}(\lambda-1)U + (1-\lambda)\left(\frac{5986}{75\tau} - \frac{699}{225\tau^3}\right)\frac{dU}{d\tau} \\
&+ (1-\lambda)\left(\frac{238}{5} + \frac{699}{225\tau^2}\right)\frac{d^2U}{d\tau^2} + \frac{6938}{225\tau}(\lambda-1)\frac{d^3U}{d\tau^3} \\
&+ \frac{238}{25}(\lambda-1)\frac{d^4U}{d\tau^4}
\end{aligned} \tag{5}$$

In Eq. (5) the emissive power has been eliminated by the use of the conservation of thermal energy. If the emissive power is specified, it will be more convenient to use the equation which eliminates the internal generation rate U.

For an infinitely long concentric cylinder with emitting and diffusively reflecting surfaces, Marshak type boundary conditions for P_5 approximation will take the following form,

$$\begin{aligned}
&15\epsilon_{w,i}\psi_0 \mp 128(2-\epsilon_{w,i})\psi_r + 210\epsilon_{w,i}\psi_{rr} \\
&- 105\epsilon_{w,i}\psi_{rrrr} = 256\epsilon_{w,i}\pi\Phi_{w,i}
\end{aligned} \tag{6a}$$

$$\begin{aligned}
&(30\epsilon_{w,i} - 35)\psi_0 \mp 256(1-\epsilon_{w,i})\psi_r + 210(2\epsilon_{w,i} - 1)\psi_{rr} \\
&\mp 512\psi_{rrr} + (525 - 210\epsilon_{w,i})\psi_{rrrr} = 512\epsilon_{w,i}\pi\Phi_{w,i}
\end{aligned} \tag{6b}$$

$$\begin{aligned}
&(30\epsilon_{w,i} - 21)\psi_0 \mp 256(1-\epsilon_{w,i})\psi_r + 210(2\epsilon_{w,i} - 3)\psi_{rr} \\
&+ 105(11 - 2\epsilon_{w,i})\psi_{rrrr} \mp 768\psi_{rrrrr} = 512\epsilon_{w,i}\pi\Phi_{w,i}
\end{aligned} \tag{6c}$$

where the + sign will be used for the inner boundary, $i=1$, and the − sign will be used for the outer boundary, $i=2$. These boundary conditions for constant heat generation rate problems can be expressed in terms of ψ_0 and the gradients of ψ_0.

Results and Discussion

Figure 2 shows heat transfer between concentric cylinders for the case of a medium in radiative equilibrium. The medium absorbs most of the energy before it reaches the outer surface, re-emitting most of it back to the inner surface; therefore, the heat transfer decreases with increasing optical thickness.

The comparisons of approximate solutions with the Monte Carlo solutions are displayed in the same figure. The Monte Carlo solution automatically accounts for the piecewise

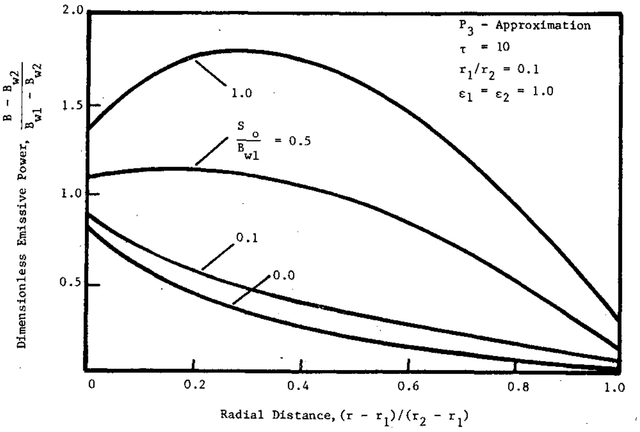

Fig. 3 Emissive power distribution between concentric cylinders, effect of heat source.

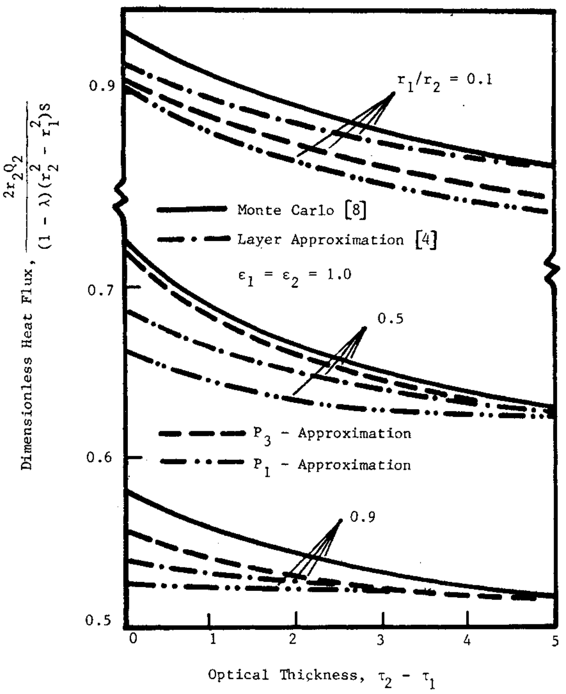

Fig. 4 Heat loss to outer cylindrical wall for medium with uniform heat source.

distribution of intensity; therefore, accurate results may be obtained. In the limiting case of zero optical thickness, the respective percentage overestimates of heat flux as found from the P_5, P_3, P_1 and diffusion approximations are 2, 5, 33, and 100% for a radius ratio of 0.5; and for a radius ratio of 0.1 the overestimates are about 35, 55, 81, and 100%. A decrease in emissivity causes a decrease in heat transfer since part of the energy emitted by the inner surface is reabsorbed because of reflection by the outer surface. For very small emissivities reflection and reabsorption become dominant, making heat exchange virtually independent of the optical thickness. The approximations give more or less equal results even for surface emissivities of 0.7. Higher orders of approximation have a negligible advantage over P_1 approximation for small surface emissivities.

The differential approximations can be used in determining the general influence of various parameters with minimum computational effort. The internal heat generation modifies the emissive power distribution, a modification easily calculated by either P_1 or P_3 approximation. Figure 3 shows the modification for a radius ratio of 0.1 and optical thickness of 10 as obtained by the P_3 approximation.[7] A constant heat

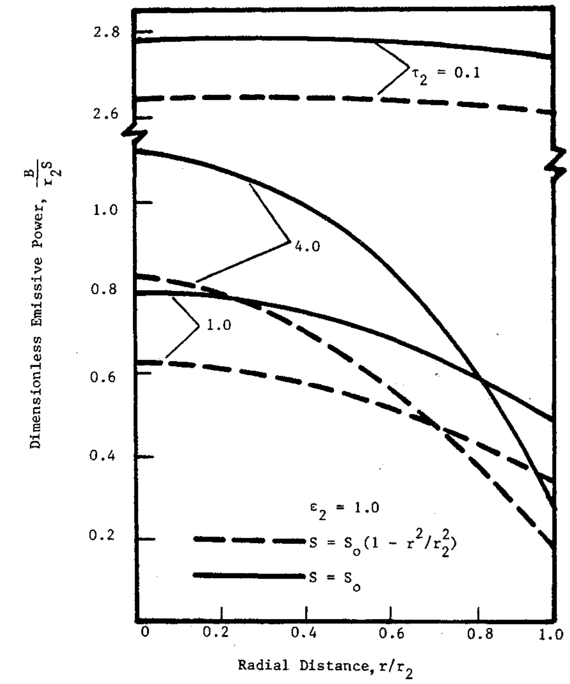

Fig. 5 Emissive power distribution in a cylinder with variable heat source.

source was assumed. From the figure, the internal heat generation which reverses the heat flow at the inner surface is given by the ratio $S(r_2 - r_1)/B_{w1}$ of 0.5 or greater. The ratios will be different for various optical thickness and radius ratio values.

The effect of the radius ratio, r_1/r_2, on heat transfer in a medium with uniform heat source is shown in Fig. 4. Surfaces were maintained at zero temperatures. For comparison purposes, results from the Monte Carlo solution[8] and layer approximation[4] are included. The maximum percentage errors of P_3, P_1, and layer approximations relative to the Monte Carlo are observed to be about 4, 8 and 5% respectively.

There are cases, for example in nuclear reactors, where the internal heat generation is a function of the radial distance. Assuming a parabolic heat source distribution, the emissive power was calculated from the P_3 approximation and results displayed in Fig. 5. The outer surface was maintained at zero temperature. The effect of a variable heat source is to decrease the emissive power and, consequently, the heat flux at the outer surface. The modification necessary to account for a variable heat source in the P_3 approximation is less complicated than the modification which would be needed in other solution methods.

The effect of isotropic scattering on the heat flux and emissive power distributions can be included as a parameter in the ordinate variable. Scattering will reduce the emissive power of the medium and the heat transfer to the walls.

For the case of isotropic scattering, zero surface temperature and a uniform heat source, the desired quantities are proportional to the scattering coefficient parameter. Therefore, it has no effect on the relative errors between the approximate and the Monte Carlo solutions.

Acknowledgment

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J80-135

On the Determination of the Polytropic Specific Heat

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Nomenclature

c_n = polytropic specific heat
c_p = specific heat at constant pressure
c_v = specific heat at constant volume
k = ratio of specific heats c_p/c_v
m = mass
n = polytropic exponent
p = pressure
Q = transferred heat
S = entropy
T = temperature

Subscripts

1 = initial condition
2 = final condition

Introduction

DURING an isobaric process, transferred heat and changes in entropy are given in differential form by

$$dQ = mc_p\,dT \tag{1}$$

and

$$dS = mc_p\,dT/T \tag{2}$$

respectively. Accuracy permitting, their integration is usually performed for constant specific heat c_p or some acceptable mean value $\bar{c}_p$. Similarly, the corresponding integrated equations for an isometric process are

$$Q = mc_v(T_2 - T_1) \tag{3}$$

and

$$S_2 - S_1 = mc_v \ln T_2/T_1 \tag{4}$$

which differ from the integrated isobaric process equations only in the specific heat factor.

In the case of a polytropic process, the mathematical development leads to transferred heat and changes in entropy expressions of the form

$$dQ = mc_v\left(\frac{k-n}{1-n}\right)dT \tag{5}$$

and

$$dS = mc_v\left(\frac{k-n}{1-n}\right)\frac{dT}{T} \tag{6}$$

respectively, where $c_v(k-n)/(1-n)$ is customarily called the polytropic specific heat, denoted by c_n, so that at least in form the equations for both transferred heat and entropy changes are similar to the preceding cases, i.e.,

$$dQ = mc_n\,dT \tag{5a}$$

and

$$dS = mc_n\,dT/T \tag{6a}$$

Here, as in the preceding cases, the desirability of a constant c_n or an acceptable mean value $\bar{c}_n$ for integration purposes is obvious.

However, while the literature abounds with tabular and graphical data for c_p and c_v, such is not the case for c_n, and while the applicable literature routinely refers to the polytropic process as the most general of the "labeled" processes—substitution of the appropriate values for n will indeed yield these processes—as special cases of the polytropic process the effect of variable n on the polytropic specific heat itself, in general, and the significance of small variations of n with respect to c_n, in specific, are neither shown nor discussed. The latter point is of no small interest since rarely, if ever, is n known exactly throughout an entire process.

Analysis

In the following, it will be shown that the expression established for the polytropic specific heat, i.e., $c_n = c_v(k-n)/1-n)$, represents a well-defined geometric figure which, once recognized, admits of rapid determination of c_n as well as the prediction of effects of uncertainties in n for any gas whose specific heat ratio k is known.

When c_n is rewritten in the nondimensionalized form $c_n/c_v = (k-n)/(1-n)$ and its functional dependence is indicated by setting $f(n) = c_n/c_v$, the relation $f(n) = (k-n)/(1-n)$ may be examined from a purely mathematical point of view. Treating n and $f(n)$ as the axes of a Cartesian coordinate system, it becomes immediately apparent that this relation exhibits two asymptotes, one at $n=1$ and the other at $f(n)=1$.

The presence of these rectangular asymptotes with common point of intersection at (1,1) seems to indicate that the graphical representation of the polytropic specific heat equation is a translated and clockwise-rotated hyperbola with center at $(n,f(n)) = (1,1)$ and an angle of rotation equal to 45 deg.

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